SEMESTRAL EXAMINATION

COMPLEX ANALYSIS
B. MATH III YEAR
I SEMESTER, 2009-2010

Notations: $U = \{z : |z| < 1\}, H(\Omega)$ is the space of holomorphic functions on the region Ω, γ^* is the range of the path γ.

Max. marks: 100 Time Limit: 3hrs

1. Prove or disprove the following:
if f is an entire function such that $\left|f(|z|^{1/3})\right| \leq 2 + 3\,|z|^{300}$ then f is a polynomial. [10]

2. Prove (in detail) that $\int\limits_{-\pi}^{\pi} \frac{e^{it}+z}{e^{it}-z}\phi(t)dt$ is a holomorphic function of z on U for any continuous function ϕ on $[-\pi, \pi]$ with $\phi(-\pi) = \phi(\pi)$. [15]

3. Find the number of zeros of the polynomial $1 - 2z^{10} + (3/4)z^n$ in U for any integer $n > 10$. [10]

4. Prove that $\prod\limits_{n=1}^{\infty} \frac{(1-\frac{1}{n^2})-z}{1-(1-\frac{1}{n^2})z}$ converges uniformly on compact subsets of U to a holomorphic function whose zeros have 1 as a limit point. [10]

5. Let f be a holomorphic function on $\mathbb{C}\backslash\{0\}$ such that f has a pole at 0 and $z^2 f(z)$ is bounded on $\{z : |z| \geq a\}$ for some positive number a . Prove that the residue of f at 0 is necessarily 0. [10]

6. Find an entire function whose real part is $1 + 2x^2 - 2y^2 + 3x^3 - 9xy^2$.[15]

7. Find all holomorphic functions f on $\mathbb{C}\backslash\{1\}$ such that f has a pole of order 3 at 1 and $\mathrm{Re}[(z-1)^3 f(z)] \geq 3$ for all z. [10]

8. Evaluate $\int\limits_{0}^{\infty} \frac{x\sin(x)}{x^4+1}dx$ by the method of residues. [20]